

Markov Modulated Punctured Autoregressive Processes for Traffic and Channel Modeling*

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Abstract

We present a new stochastic process called the *punctured* autoregressive (AR) process, and use it to model both the variable bit rate (VBR) video traffic and the wireless channel dynamics. To model the VBR video traffic, we propose to use punctured autoregressive processes modulated by a doubly Markov process. The doubly Markov process models the state of each video frame while the autoregressive processes describe the number of bits of each frame at each state. The punctured autoregressive process considers the timing information between frames of the same state and thus gives better modeling performance. The model captures the long-range dependency (LRD) characteristics as well as the short-range dependency (SRD) characteristics of the video traffic. Queuing behavior of the punctured autoregressive process is also closer to the real video traffic than the conventional autoregressive process. In addition to video traffic modeling, we also apply the same model to wireless channel dynamics. The channel error rate is modeled as a single Markov modulated punctured autoregressive process. The synthetic channel error rate generated by the punctured autoregressive process performs closer to the real channel error rate than the one generated by the conventional autoregressive process.

I. Introduction

To both the video service providers and the network designers, it is important to have a good model for the video traffic. A good model for video traffic allows for better admission control, scheduling, network resource allocation policies, etc., that guarantee a desired quality of service (QoS) as well as a better utilization of the network resources. A good model captures essential characteristics of the real video traffic. The synthetic trace generated by such a model can be used to test the network performance under a certain, for example, admission control policy. Therefore, the network designers can design a network that is more friendly to the video traffic and thus delivers a better video service.

In addition to the video traffic modeling, it is also crucial to model the wireless channel dynamics. In wired networks, the transmission is nearly error free. However, in the wireless network, the channel error rate fluctuates depending on the noise, distance, speed of the mobile unit, multi-path interference, etc. A good model of the wireless channel dynamics, in particular the channel error rate in this paper, can be used to provide a better error control mechanism for the data packets as well as a better wireless network design. For example, we can test different Automatic Repeat Request (ARQ) protocols with synthetic channel errors generated by the channel modeler to select an ARQ scheme that suits the best.

Because of the importance of both the variable bit rate (VBR) video traffic modeling and the wireless channel dynamic modeling, many models have been proposed. To evaluate different models, there are generally three criteria to be taken into account— (1) A good model should capture the statistical properties of the real trace. A trace is defined as a sequence of data we intend to model. In the case of VBR video traffic modeling, a trace is a sequence of numbers, each represents the number of bits to encode each Group of Blocks (GOB)/frame/Group of Pictures (GOP). In the case of wireless channel modeling, a trace is a sequence of numbers, each represents the channel bit error rate (BER) at different time instant. The statistical properties should include those that are related to the long-range dependency (LRD) of the trace as well as those that are related to the short-range dependency (SRD) of the trace [1].

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(2) The synthetic video trace should be similar to the real video trace in terms of the queuing behavior. Likewise, the synthetic trace of the wireless channel should be similar to the real trace in terms of the QoS behavior. (3) The model should be simple and easy to be analyzed. Related discussion on how to evaluate the performance of the models can be found in [2]-[4].

Video traffic modeling is challenging in several aspects. First, video is usually encoded with VBR to be adaptive to the video content. Second, depending on different coding schemes, the video trace has very different properties. Popular video coding schemes include H.263 [5] and MPEG-4 [6]. The video frame can be Intra (I), Predictive (P), or Bidirectionally predictive (B) encoded. The GOP structure, which is defined as frames enclosed by two I frames including the leading I frame, can be fixed or dynamic. A fixed GOP structure that is commonly used is IBBPBBPBBPBB.

Existing work for video traffic modeling includes DAR [7], which fails to capture the LRD property of the video traffic. Models such as [8]-[11] are constrained by a fixed GOP structure. Non-statistical methods as [12][13] are usually more difficult to analyze. The wavelet-based model [14] and the autoregressive process (AR) based model [15] do not capture the dynamic nature of the video traffic nicely. In general, it is preferred to use a Markov chain like process to model the dynamic nature of the video traffic. Models such as [16] are too complex. We propose to build a model based on the work done by [17], which models the video traffic as a doubly Markov process with AR processes inside each Markov state. However, the model of [17] does not use the timing information between frames of the same Markov state. In this paper, we explicitly use the timing information between frames of the same Markov state and refer to the new model as a “*doubly Markov process modulated punctured AR process*”. It is shown that the proposed model outperforms the model of [17] in terms of statistics, both SRD and LRD, queuing behavior, and has the same complexity in terms of the number of model parameters.

Conventional channel analyses model the channel statistics with physical layer parameters such as the signal to noise ratio (SNR), speed of the mobile unit, etc. Most of the models concentrate on matching the probability distribution function (pdf) of the synthetic trace with the real trace of the channel and do not match the LRD property such as the Hurst parameter [3]. In addition, a channel model with a perfect statistical property match to the real channel does not guarantee similar behavior to the packets transmitted on top of which. It is more important to capture the behavior of the packets sending over the channel. Some related models based on Markov process can be found in [18]. More related work in modeling the channels can be found in [19][20]. We propose in this paper a “*Markov process modulated punctured AR process*” to model the error rate of the channel. It is shown that the punctured AR process performs better than the conventional AR process.

This paper is organized as follows. Section II introduces the core concept of this paper—“punctured AR process” as opposed to the conventional AR process. In Section III, we present the doubly Markov process modulated punctured AR process as well as the conventional non-punctured scheme. Both models are shown side by side with performance comparisons. In Section IV, we apply the punctured AR process to model the wireless channel. The non-punctured scheme is also presented and compared. Section V concludes our work.

II. Punctured Autoregressive Modeling

A conventional autoregressive (AR) process x_n is defined as follows:

$$x_n = \mathbf{r}x_{n-1} + \mathbf{s}\sqrt{1 - \mathbf{r}^2}e_n, \quad n = 1, 2, \dots \quad (1)$$

where n is the time index of the process, \mathbf{r} describes the dependency of the sample at time n with the previous sample at time $n-1$, \mathbf{s}^2 is the variance of the process x_n , and e_n is a Gaussian random variable with mean 0 and variance 1 to characterize the random nature of the process.

Consider the case where more than one AR processes are interleaved together such as shown in Figure 1 (a). At time instant 1, AR process x_n takes place; at time instant 2, AR process y_n takes place; and so on.

Conventional method to train two sets of AR parameters of sequences x_n and y_n is by splitting the single process in Figure 1 (a) to two separate processes as shown in Figure 1 (b) and Figure 1 (c). Each one of the processes represents the training sequence of x_n or y_n regardless of the time index associated with each sample. For example, the sequence in Figure 1 (b) is used as if samples are

$$\tilde{x}_1 \tilde{x}_2 \tilde{x}_3 \tilde{x}_4 \tilde{x}_5 \tilde{x}_6 \tilde{x}_7 \tilde{x}_8 \tilde{x}_9 \quad (2)$$

where $\tilde{x}_1 = x_1$, $\tilde{x}_2 = x_3$, and so on.

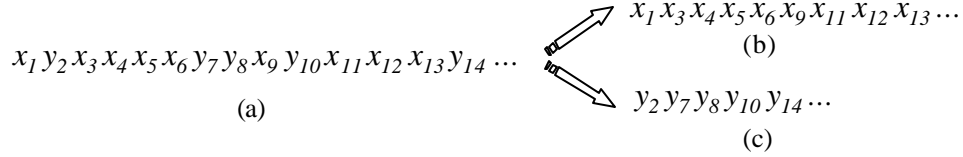


Figure 1. Two interleaved autoregressive processes x_n and y_n : (a) the interleaved process; (b) autoregressive process x_n ; (c) autoregressive process y_n .

To synthesize samples using this model, two separate AR processes are generated with parameters trained by \tilde{x}_n and \tilde{y}_n . Synthetic samples generated by parameters trained by \tilde{x}_n are taken one after the other to form the final synthetic process. Synthetic samples generated by parameters trained by \tilde{y}_n are formed in a similar way. In brief, “no synthetic samples are skipped”.

It can be seen that the conventional method does not take into account the timing information of the two processes in both the training and synthesis stages. To consider the timing information, we propose to train and synthesize the AR processes as follows (Figure 2). First split the single process in Figure 2 (a) to two separate processes as shown in Figure 2 (b) and Figure 2 (c). Notice that the timing information is utilized while leaving the sample of x_n blank if at some particular time instance the original process is with the other process y_n (Figure 2 (b)). Similarly, the sample of y_n is blank if at some time instance the original process is with the other process x_n (Figure 2 (c)).

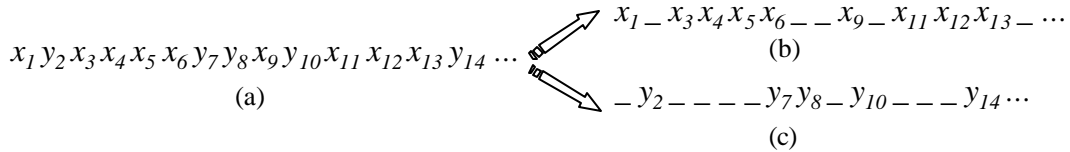


Figure 2. Two interleaved autoregressive processes x_n and y_n : (a) the interleaved process; (b) punctured autoregressive process x_n ; (c) punctured autoregressive process y_n .

To train the punctured AR processes, we find the value $\tilde{\mathbf{r}}_x$ of the process $\tilde{x}_1 \tilde{x}_2 \tilde{x}_3 \tilde{x}_4 \tilde{x}_5 \tilde{x}_6 \tilde{x}_7 \tilde{x}_8 \tilde{x}_9$. We also construct the histogram of the sample spacing. For example in Figure 2 (b), the histogram has values [5, 2, 1] at spacing [1, 2, 3]. Recall that in the simplest case where all the samples are adjacent to each other, the histogram has values [8, 0, 0] at spacing [1, 2, 3]. The \mathbf{r}_x of the process in Figure 2 (b) can be found by solving the following equation:

$$\frac{5\mathbf{r}_x^1 + 2\mathbf{r}_x^2 + 1\mathbf{r}_x^3}{(5 + 2 + 1)} = \tilde{\mathbf{r}}_x \quad (3)$$

The value of \mathbf{r}_y in Figure 2 (c) can be solved in the same way.

To synthesize samples using this model, two separate AR processes are generated by r_x and r_y . The final synthetic process is formed by some means of multiplexing/modulation of the two synthetic processes. Samples are not taken one after the other but with consideration of how separate the samples of the same type are. In brief, “some synthetic samples are skipped”. Punctured AR processes can be modulated by different processes. In this paper, we specifically consider the Markov process.

III. Variable Bit Rate Video Traffic Modeling

1. Doubly Markov modulated AR processes: *Punctured* and *Conventional*

Before we proceed to the proposed model for VBR video traffic, let us briefly describe the method proposed by [17] with which we will compare (Figure 3). The model comprises of two layers of Markov process. Without loss of generality, we consider two frame types, I and P, in this paper. The doubly Markov process models I and P frame transitions, as well as different frame activities. The outer Markov process describes how I and P frames transit. The frame type can be further categorized into different activity levels. Frames of higher activity level consume more number of bits, while frames of lower activity level consume less number of bits. The inner Markov process describes how the frames of different activity levels transit. This model is not constrained by a fixed GOP structure. There are six states in total, namely, I frame in high activity level, I frame in medium activity level, I frame in low activity level, P frame in high activity level, P frame in medium activity level, and P frame in low activity level.

Each state is modeled as an AR process with different AR parameters. We can consider this as a slightly more complex process than the one described in Figure 1 of Section II. There are two states in Figure 1 (a) of Section II while there are six states in this model. The AR parameters are trained in the way described in Figure 1 of Section II in a conventional non-punctured manner. The synthetic trace is generated by procedures described in Section II as well.

To simplify the model, the inner Markov process of I frames is characterized by initial probabilities of three activity levels only. Since I frames are usually far apart in a video sequence, they do not need to modeled as a Markov process. Such simplification will have similar performance as the full model. We call this model Method 1, as shown in Figure 3 (a), for later discussion.

We propose to model the VBR video traffic as a doubly Markov modulated punctured AR process. This new model explicitly considers the timing information between two frames of the same state. Again, this is a slightly more complex process than the one described in Figure 2. There are two states in Figure 2 (a) of Section II while there are six states in this model. The AR parameters are trained in the way described in Figure 2 of Section II in a punctured manner. The synthetic trace is generated by procedures described in Section II as well. We call this model Method 2, as shown in Figure 3 (b), for later discussion.

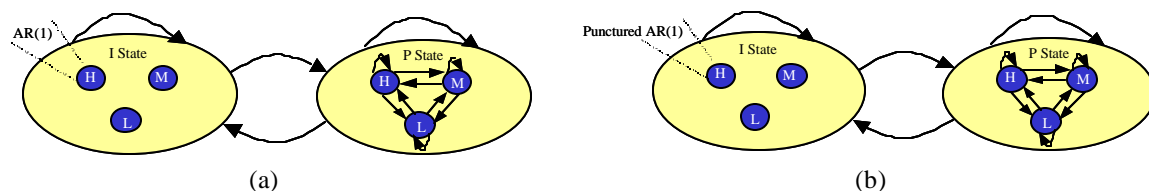


Figure 3. Models for VBR video traffic— (a) Method 1: Doubly Markov modulated AR process; (b) Method 2: Doubly Markov modulated punctured AR process.

2. Experiment

We now compare the performance of the non-punctured Method 1 with the proposed punctured Method 2. To evaluate the performance of the models, we consider four performance metrics: (1) first order statistics by means of the quantile-quantile (Q-Q) plot; (2) second order statistics by means of the auto-correlation function (ACF); (3) LRD property by means of the Hurst parameter from the range/standard deviation (R/S) plot; and (4) queuing behavior by means of the packet loss rate and the queuing delay. Definitions of the performance metrics can be found in [2]-[4].

The experiment setting is as follows. Two different types of TV programs are recorded: “news” as shown in Figure 4 (a) and “talk show” as shown in Figure 4 (b). The two TV programs are encoded using video compression codec H.263 to generate the real video traces. Both of them are encoded with frame rate of 15 frames/sec and with duration of 30 minutes each. The video trace of the clip “news” is shown in Figure 5 (a) and (b) with different scales. The video traces are then fed into both models: Method 1 and Method 2 to generate synthetic traces. The performances of the two models are evaluated.



Figure 4. Test videos: (a) news; (b) talk show

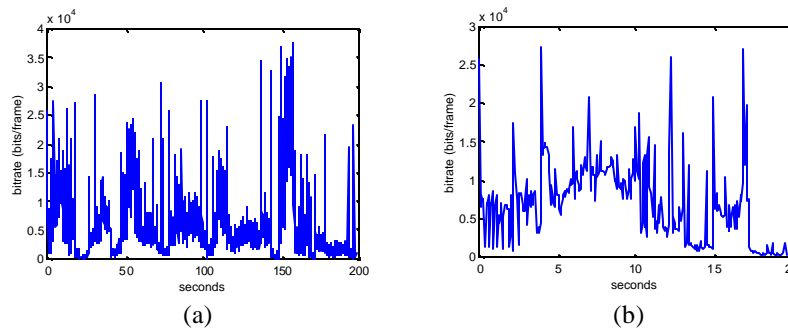


Figure 5. Sample traces from the TV program “news”: (a) a 200 second trace; (b) a 20 second trace.

The performance comparison is summarized in Table 1. MSE refers to mean square error compared to the real trace. It can be seen that the proposed Method 2 outperforms Method 1 in all five aspects. The MSE improvement is computed by $|(MSE \text{ of Method 2}) - (MSE \text{ of Method 1})| / (MSE \text{ of Method 1})$. Detailed discussion associated with each performance metric will be presented later.

Table 1 Summary of performance comparison between modeling methods Method 1 and Method 2

	MSE of Q-Q plot	MSE of ACF	Hurst parameter	MSE of packet loss rate	MSE of queuing delay
Real	/	/	0.2530	/	/
Method 1	1.2048e+7	1.1126e+14	0.1538 (98e-4 in SE)	9.7846e-4	4.6553e-4
Method 2	0.1747e+7	0.4054e+14	0.2725 (3.8025e-4 in SE)	9.6003e-4	3.4044e-4
MSE improvement	85.50%	63.56%	96.12%	1.88%	26.87%

Figure 6 (a)(b) shows the performance of both models in terms of first and second order statistics. The first order statistics in Figure 6 (a) is shown by the Q-Q plot. The Q-Q plot is constructed by a pair of cumulative distribution functions (CDF). The closer one CDF is to the other CDF in one pair, the more the curve will look like a straight line $y = x$. In Figure 6 (a), a dotted straight line is plotted as a reference. We have two pairs of CDF to compare: the synthetic trace by Method 1 with respect to the real video trace and the synthetic trace by Method 2 with respect to the real video trace. The dashed-dotted curve in Figure 6 (a) refers to the first pair. The solid curve in Figure 6 (a) refers to the second pair. It is shown that the curve of Method 2 is closer to the reference dotted straight line than the curve of Method 1.

The second order statistics in Figure 6 (b) is shown by the ACF. The dotted curve in Figure 6 (b) refers to the ACF of the real video trace. The dashed-dotted curve in Figure 6 (b) refers to the ACF of the synthetic trace by Method 1. The solid curve in Figure 6 (b) refers to the ACF of the synthetic trace by Method 2. It is shown that the curve of Method 2 is closer to the reference curve than the curve of Method 1.

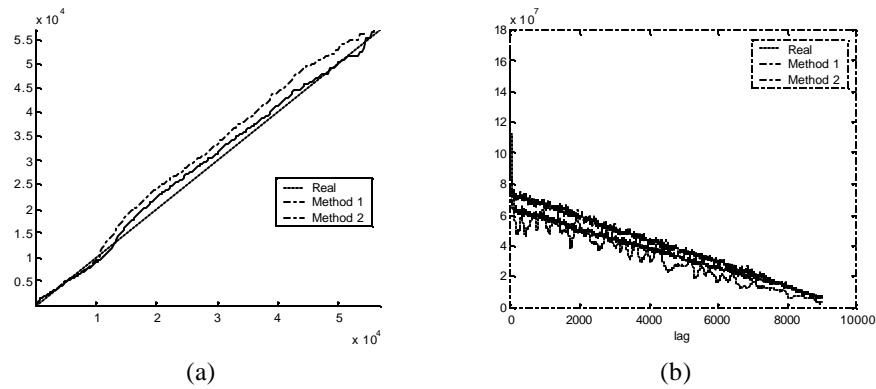


Figure 6. First and second order statistics of the synthetic traces generated by Method 1 and Method 2 with respect to the real video trace of the clip “news”. (a) First order statistics: Q-Q plot; (b) Second order statistics: ACF.

The LRD property in Figure 7 is shown by means of the Hurst parameter, which is the slope of the linear regression line of the points in a R/S plot. Figure 7 (a) shows the R/S plot of the real video trace. The linear regression line is shown as a dotted line. Figure 7 (b) shows the R/S plot of the synthetic trace by Method 1. Figure 7 (c) shows the R/S plot of the synthetic trace by Method 2. It is shown that the synthetic trace by Method 2 has closer Hurst parameter value to the real video trace than the synthetic trace by Method 1.

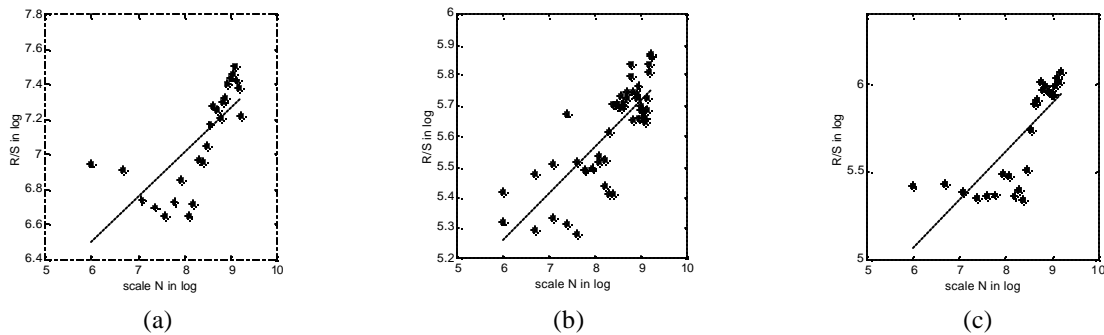


Figure 7. LRD properties of three traces by Hurst parameter from the R/S plots: (a) real video trace; (b) synthetic trace by Method 1; (c) synthetic trace by Method 2

The queuing behavior of the traces is evaluated by means of the packet loss rate and the queuing delay. Packet loss rate and queuing delay are measured at different drain rates and buffer sizes. The network is a leaky bucket with drain rate R and time to drain M/R , where M is the buffer size. The queuing performance of the real trace is shown in Figure 8. The queuing performances of the synthetic traces by both Method 1 and Method 2 have similar look. However, the synthetic trace by Method 2 has smaller MSE in both the packet loss rate and the delay than the synthetic trace by Method 1 (Table 1).

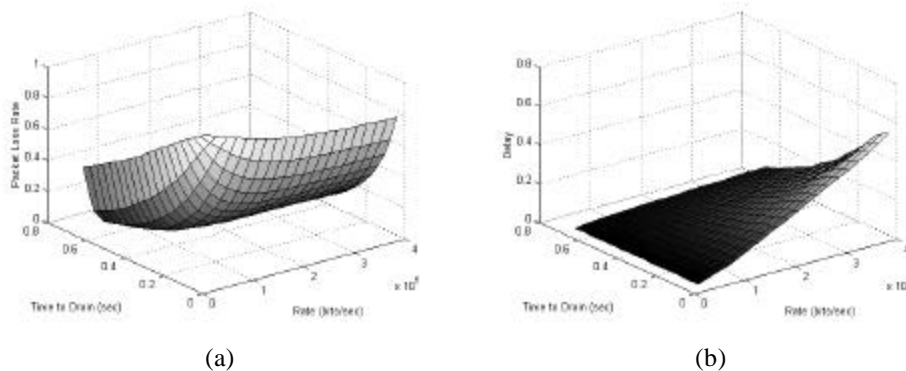


Figure 8. Queuing behavior of the real video trace: (a) packet loss rate; (b) queuing delay.

IV. Wireless Channel Modeling

In this paper, we model in particular the bit error rates (BER) of the channel at different time instances. Unlike other channel modeling work, which models the channel with information such as the SNR of the received signal, speed of the mobile unit, channel modulation and coding scheme, etc., we model the channel solely from the trace of the bit error rates. To evaluate the models, we not only examine the statistics of the synthetic traces but also the queuing behaviors of the traces with ARQ as the error control scheme. We present two models with variations in the underlying AR processes: punctured and non-punctured.

1. Markov modulated AR processes: *Punctured* and *Non-Punctured*

The BER at any time instance can be categorized as high, medium, or low. The three error rate states: high, medium, and low, are considered as three states in a Markov process with certain initial probabilities and transition probabilities. At each state, the BER is modeled as an AR process. There are three AR processes modulated together by a single Markov process. The Markov modulated non-punctured and punctured AR processes are illustrated in Figure 9 (a)(b). We call the conventional non-punctured Markov modulated AR process Method 1 and proposed Markov modulated punctured AR process Method 2. The training and synthesis procedures are described in Section II with number of AR processes three.

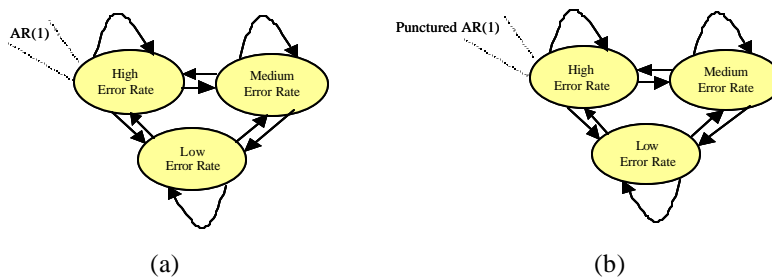


Figure 9. Models for the channel— (a) Method 1: Markov modulated AR process; (b) Method 2: Markov modulated *punctured* AR process

2. Experiment

A sample trace of BER in time is shown in Figure 10, which is a Rayleigh fading channel [21].

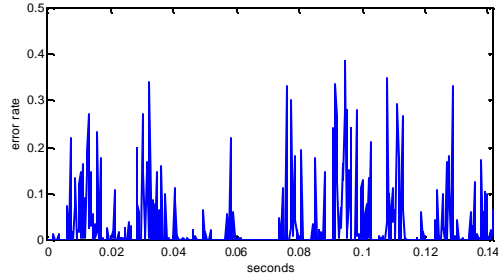


Figure 10. Sample trace of the wireless channel in BER over time.

The performance comparison is summarized in Table 1. It can be seen that the proposed Method 2 outperforms Method 1 in the Hurst parameter and the queuing behavior. Method 2 does not perform as well as Method 1 in terms of first and second order statistics. Since we are more interested in the wireless network performance, we should concentrate more on the queuing behavior. Detailed discussions associated with each performance metric will be presented later.

Table 2 Summary of performance comparison between modeling methods Method 1 and Method 2

	MSE of Q-Q plot	MSE of ACF	Hurst parameter	MSE of packet loss rate	MSE of queuing delay
Real			0.0727		
Method 1	0.0071	2.7035e-7	0.1349 (0.0039 in SE)	9e-4	13e-4
Method 2	0.0276	15.882e-7	0.1173 (0.0020 in SE)	4.1315e-4	3.5550e-4
MSE improvement	-288.73%	-487.46%	48.72%	78.26%	72.65%

Figure 11 shows the first and second order statistics of the synthetic traces generated by Method 1 and Method 2. The straight dotted line in Figure 11 (a) is the reference. The dashed-dotted curve in Figure 11 (a) refers to the first pair of the synthetic trace by Method 1 respect to the real trace. The solid curve in Figure 11 (a) refers to the second pair of the synthetic trace by Method 2 respect to the real trace. It is shown that the curve of the synthetic trace generated by Method 2 strays further away from the reference line than the curve of the synthetic trace generated by Method 1.

The dotted curve in Figure 11 (b) refers to the ACF of the real trace. The dashed-dotted curve in Figure 11 (b) refers to the ACF of synthetic trace by Method 1. The solid curve in Figure 11 (b) refers to the ACF of synthetic trace by Method 2. In Figure 11 (b), both ACF curves of Method 1 and Method 2 are close to the ACF curve of the real trace.

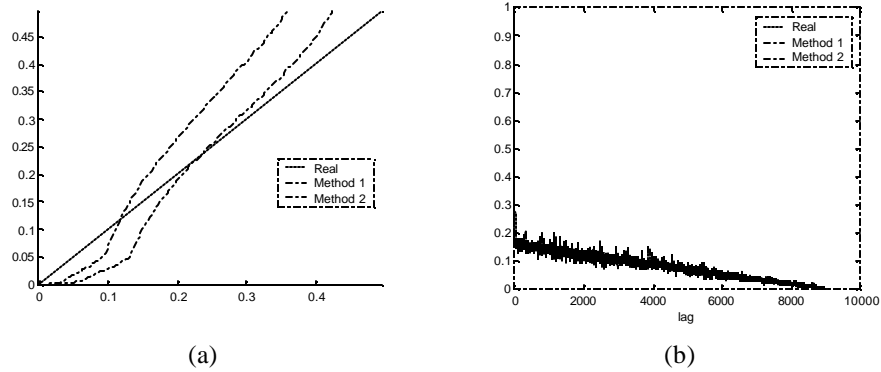


Figure 11. First and second order statistics of synthetic traces generated by Method 1 and Method 2 with respect to the real trace of the wireless channel. (a) First order statistics by Q-Q plot; (b) Second order statistics by ACF.

Figure 12 (a) shows the R/S plot of the real trace. The linear regression line is shown as the dotted line. Figure 12 (b) shows the R/S plot of trace by Method 1. Figure 12 (c) shows the R/S plot of trace by Method 2. It is shown that the synthetic trace by Method 2 has closer Hurst parameter value to the real trace than the synthetic trace by Method 1.

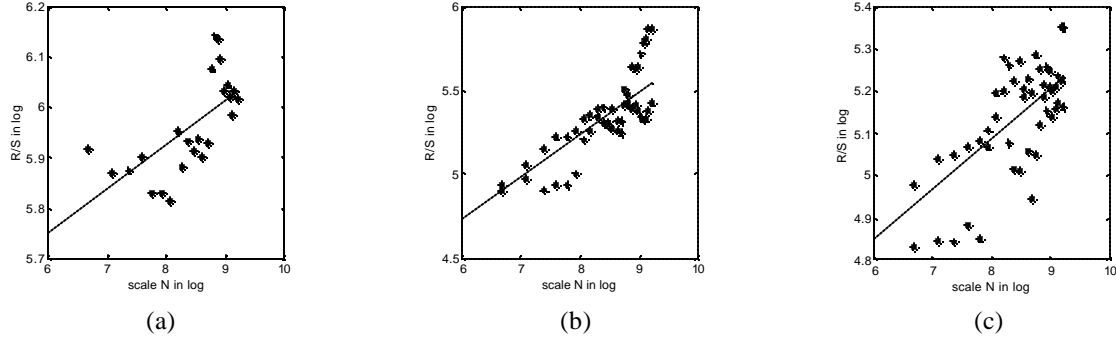


Figure 12. R/S plots of three traces: (a) real trace; (b) synthetic trace by Method 1; (c) synthetic trace by Method 2.

If ARQ is applied as the error control mechanism, we can derive the data packet service rate of the packet [19][20]. The packet error rate PER is defined as:

$$PER = 1 - [1 - BER]^n \quad (4)$$

where n is the size of a packet in bits. The data packet service rate is then:

$$R_c = [1 - PER]m \quad (5)$$

where m^{-1} is the frame interval. Note that the frame here does not refer to the video frame (Figure 13).

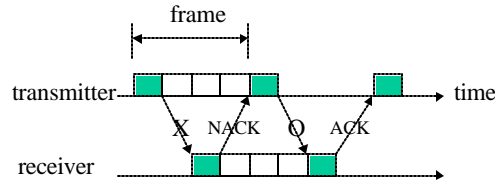


Figure 13. ARQ frame structure.

We can further analyze the queuing behavior of the data packet service rate derived from the BER trace. The queuing performance of the real trace is shown in Figure 14. The queuing performances of the synthetic traces of both Method 1 and Method 2 have similar look. However, the synthetic trace by Method 2 has smaller MSE in both the packet loss rate and the delay than the synthetic trace by Method 1 (Table 2).

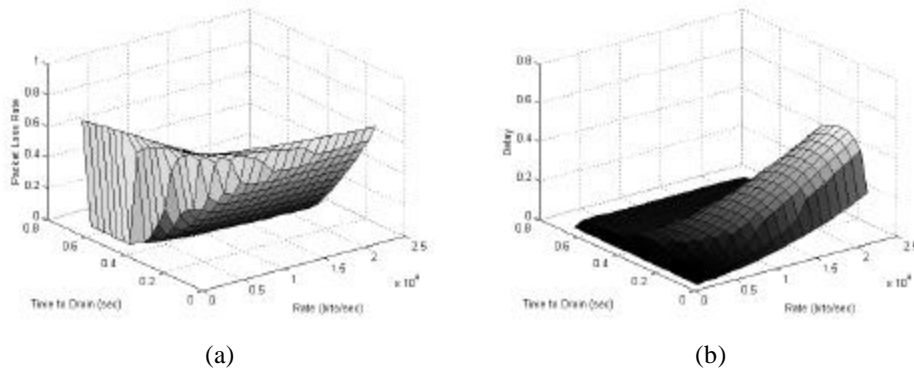


Figure 14. Queuing behavior of the real trace of the wireless channel: (a) packet loss rate; (b) queuing delay.

V. Conclusion

We proposed a new punctured AR processes to model both the video traffic and the wireless channel. The punctured AR processes are modulated by Markov processes. The punctured AR processes explicitly consider the timing information between samples of each state. Thus, it outperforms the conventional approach in VBR video traffic modeling as well as the wireless channel modeling. A good set of performance metrics are experimented showing the novelty of the proposed model in different aspects, especially in the queuing behavior.

VI. Reference

- [1] Heejume Ahn, Jae-Kyoon Kim, Song Chong, Bara Kim, and Bong Dae Choi, "A Video Traffic Model based on the Shifting-Level Process: the Effects of SRD and LRD on Queueing Behavior", *INFOCOM 2000*, pages 1036-1045.
- [2] D. M. Lucantoni, M. F. Neuts, and A. R. Reibman, "Method for Performance Evaluation of VBR Video Traffic Models", *IEEE/ACM Transactions on Networking*, Vol. 2, Issue 2, pages 176-180, April 1994.
- [3] O. Rose, "Statistical Properties of MPEG Video Traffic and Their Impact on Traffic Modeling in ATM Systems", *Proceedings of 20th Conference on Local Computer Networks*, pages 397-406, 1995.
- [4] P. R. Jelenkovic, A. A. Lazar, and N. Semret, "The Effect of Multiple Time Scales and Subexponentiality in MPEG Video Streams on Queueing Behavior", *IEEE Journal on Selected Areas in Communications*, Vol. 15, No. 6, pages 1052-1071, August 1997.
- [5] Video Coding for Low Bit Rate Communication, ITU-T Recommendation H.263 Version 2, Jan. 1998.
- [6] Motion Pictures Experts Group, "Overview of the MPEG-4 standard", ISO/IEC JTC1/SC29/WG11 N2459, 1998.
- [7] D. Heyman, A. Tabatabai, and T. V. Lakshman, "Statistical Analysis and Simulation Study of Video Conference Traffic in ATM Networks", *IEEE Transactions on Circuits and Systems for Video Technology*, Vol. 2, No. 1, March 1992.
- [8] Xinya Wang, Souhwan Jung, and J. S. Meditch, "VBR Broadcast Video Traffic Modeling- A Wavelet Decomposition Approach", *GLOBECOM 97*, pages 1052-1056, 1997.
- [9] Zhang Yongqiang, Chen Xinlin, Xiong Bingqun, "A New VBR Model of MPEG Streams on ATM Networks", *Proceedings of 1998 International Conference on Communication Technology*, 1998.
- [10] A. M. Dawood and M. Ghanbari, "Content-Based MPEG Video Traffic Modeling", *IEEE Transactions on Multimedia*, Vol. 1, pages 77-87, March 1999.
- [11] H. Liu, N. Ansari, and Y. Q. Shi, "The Tale of a Simple Accurate MPEG Video Traffic Model", *ICC 2001*, pages 1042-1046.
- [12] Xiong Simin, Liang Jinsong, Yang Zhimin, and Lei Zhengming, "Video Traffic Modeling Based on RBF Networks", *Proceedings of 1998 International Conference on Communication Technology*, 1998.
- [13] Q. Liang, J. M. Mendel, "MPEG VBR Video Traffic Modeling and Classification Using Fuzzy Techniques", *IEEE Transactions on Fuzzy Systems*, Vol. 9, No. 1, pages 183-193, February 2001.
- [14] S. Ma, C. Ji, "Modeling Video Traffic Using Wavelets", *ICC 98*, pages 559-562, 1998.
- [15] Derong Liu, E. I. Sara, and Wei Sun, "Nested Auto-regressive Processes for MPEG-Encoded Video Traffic Modeling", *IEEE Transactions on Circuits and Systems for Video Technology*, Vol. 11, No. 2, pages 169-183, February 2001.
- [16] M. M. Krunz, A. M. Makowski, "Modeling Video Traffic using M/G/ ∞ Input Processes: A Compromise between Markovian and LRD Models", *IEEE Journals on Selected Areas in Communications*, Vol. 16, Issue 5, pages 733-748, 1998.
- [17] Deepak S. Turaga and Tsuhan Chen, "Hierarchical Modeling of Variable Bit Rate Video Sources", *Packet Video 2001*.
- [18] William Turin and M. Mohan Sondhi, "Modeling Error Sources in Digital Channels", *IEEE Journal on Selected Areas in Communications*, Vol. 11, No. 3, pages 340-347, April 1993.
- [19] Young Yong Kim and San-Qi Li, "Capturing Important Statistics of A Fading/Shadowing Channel for Network Performance Analysis", *IEEE Journal on Selected Areas in Communications*, Vol. 17, No. 5, pages 888-901.
- [20] Young Yong Kim and San-Qi Li, "Modeling Multipath Fading Channel Dynamics for Packet Data Performance Analysis", *Wireless Networks*, Vol 6, pages 481-492, Dec. 2000.
- [21] T. S. Rappaport, *Wireless Communications: Principles and Practice*, Prentice Hall 1996.